Finding Similar Items:

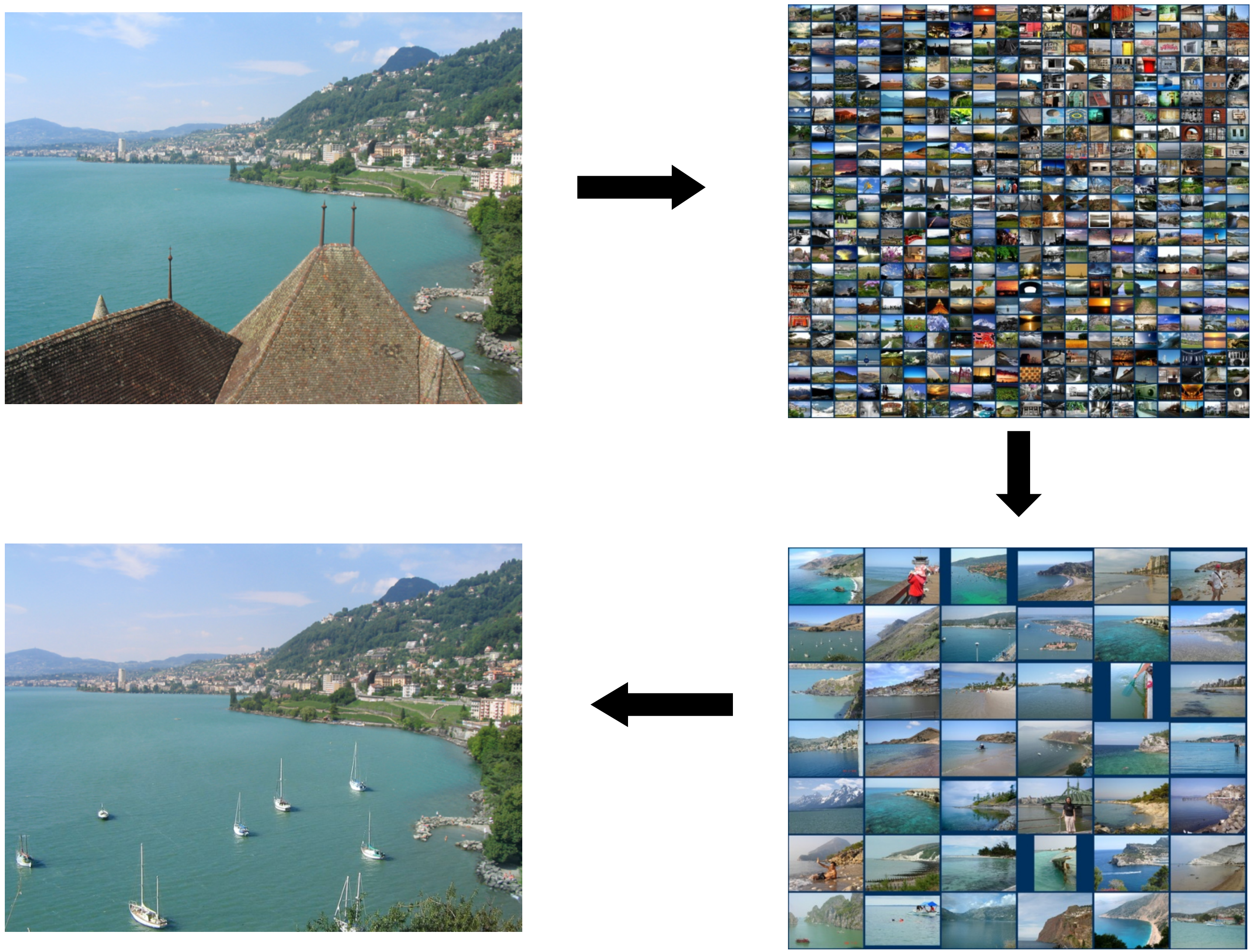
Big Data Analytics CSCI 4030

New thread: High dimensionality data

High dim data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Filtering data streams	SVM	Recommender systems
Clustering	Network Analysis	Web advertising	Decision Trees	Association Rules
Dimensionality reduction	Spam Detection	Queries on streams	Perceptron, kNN	Duplicate document detection

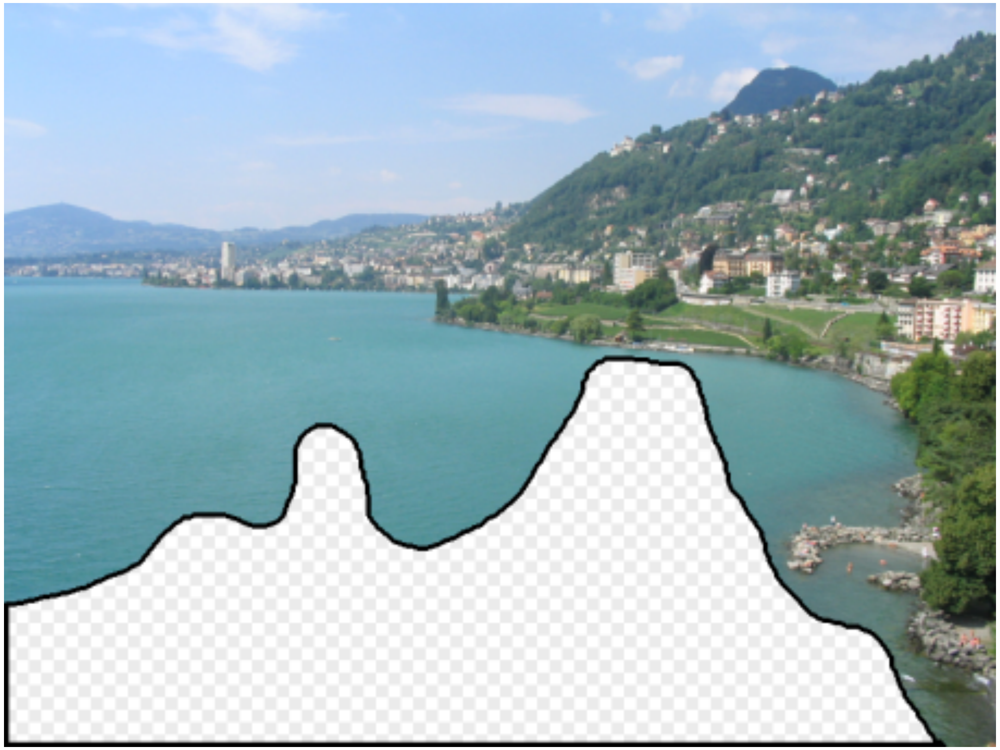
[Hays and Efros, SIGGRAPH 2007]

Scene Completion Problem



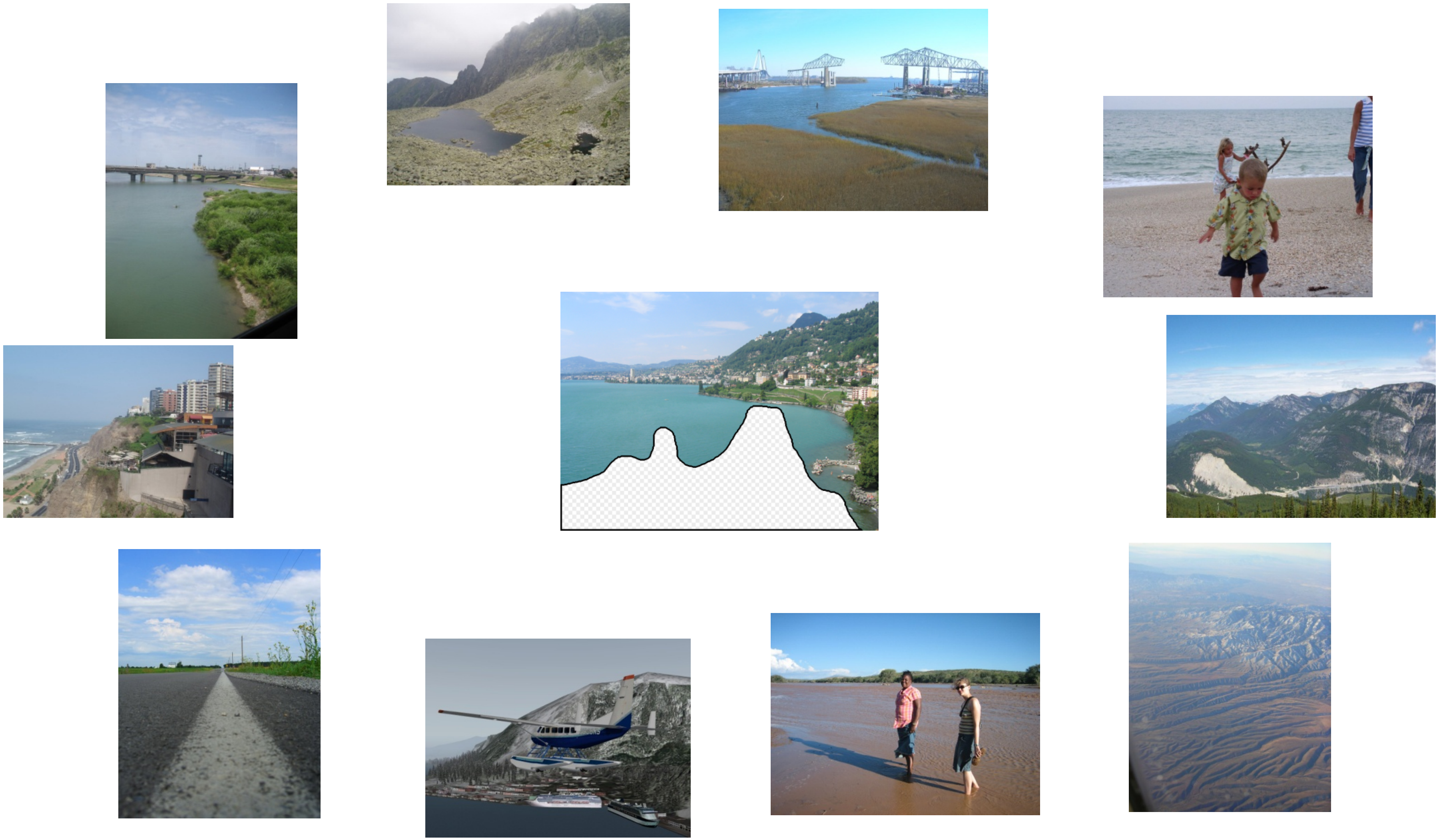
[Hays and Efros, SIGGRAPH 2007]

Scene Completion Problem



[Hays and Efros, SIGGRAPH 2007]

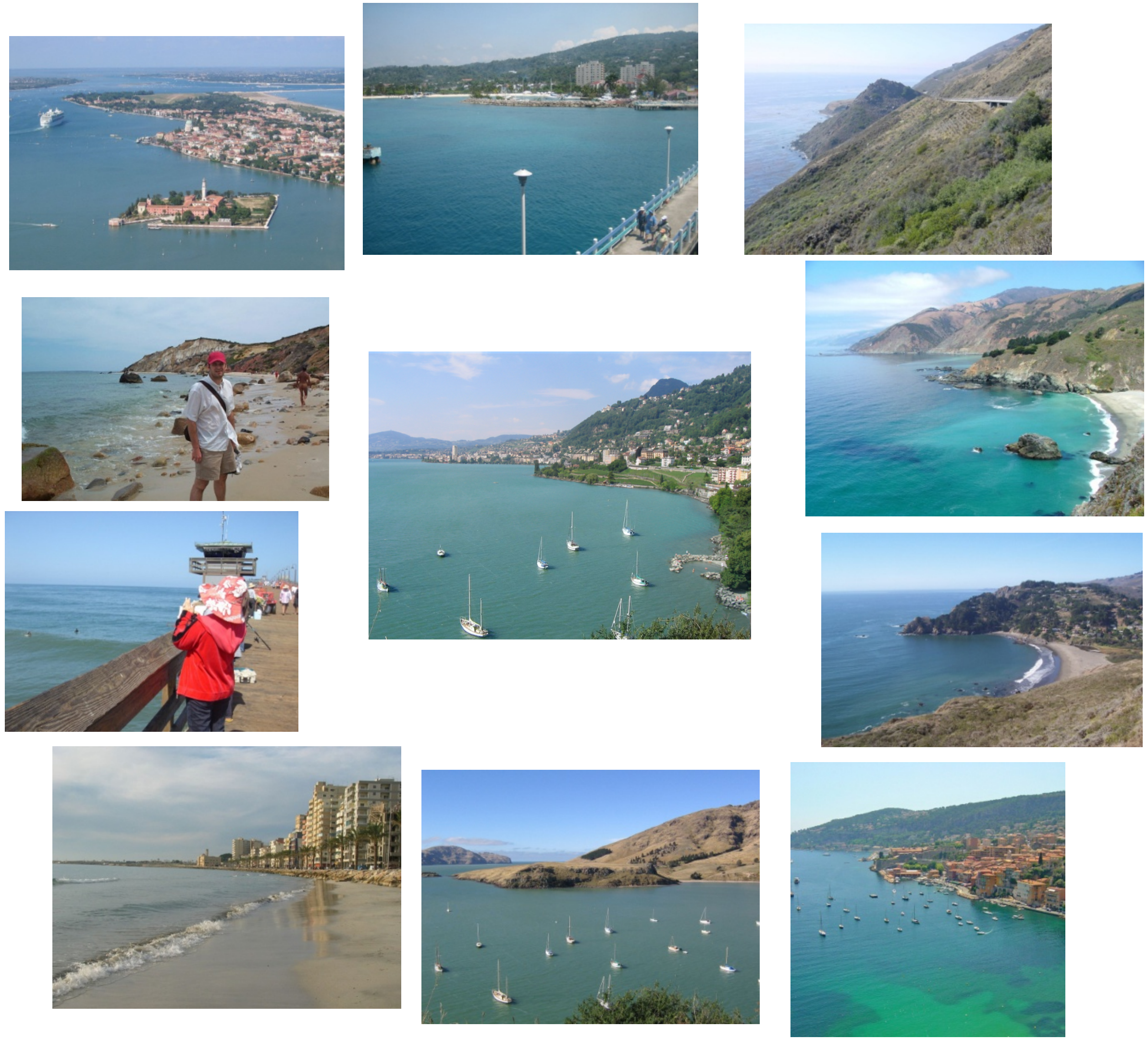
Scene Completion Problem



10 nearest neighbors from a collection of 20,000 images

[Hays and Efros, SIGGRAPH 2007]

Scene Completion Problem



10 nearest neighbors from a collection of 2 million images

A Common Metaphor

- **Many problems can be expressed as finding "similar" sets:**
  - **Find near-neighbors in high-dimensional space**
- **Examples:**
  - **Pages with similar words**
    - For duplicate detection (Mirror Pages, Plagiarism)
  - **Customers who purchased similar products**
    - Online Purchases (Amazon)

Problem for Today's Lecture

- **Given: High dimensional data points $x_1, x_2, \ldots$**
  - **For example:** Image is a long vector of pixel colors

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 2\ 1\ 0\ 2\ 1\ 0\ 1\ 0]$$

- **And some distance function $d(x_1, x_2)$**
  - Which quantifies the "distance" between x_1 and x_2
- **Goal:** Find **all pairs of data points** (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \leq s$
- **Note:** Naïve solution would take $O(N^2)$ ☹ where N is the number of data points
- **MAGIC: This can be done in $O(N)$!! How?**

Main Idea

Today's lecture: Find pairs of similar docs

Main idea: Candidates

-- Pass 1: Take documents and hash them to buckets such that documents that are similar hash to the same bucket

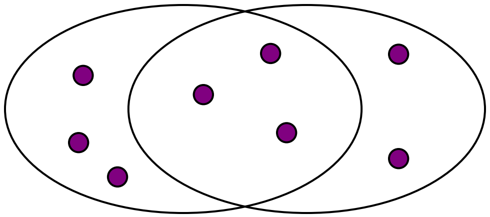
-- Pass 2: Only compare documents that are **candidates** (i.e., they hashed to a same bucket)

Benefits: Instead of $O(N^2)$ comparisons, we need $O(N)$ comparisons to find similar documents

Finding Similar Items

Distance Measures

- **Goal: Find near-neighbors in high-dim. space**
  - We formally define "near neighbors" as points that are a "small distance" apart
- For each application, we first need to define what "**distance**" means
- **Today: Jaccard distance/similarity**
  - The **Jaccard similarity** of two **sets** is the size of their intersection divided by the size of their union: $\boldsymbol{sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|}$
  - **Jaccard distance:** $\boldsymbol{d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|}$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

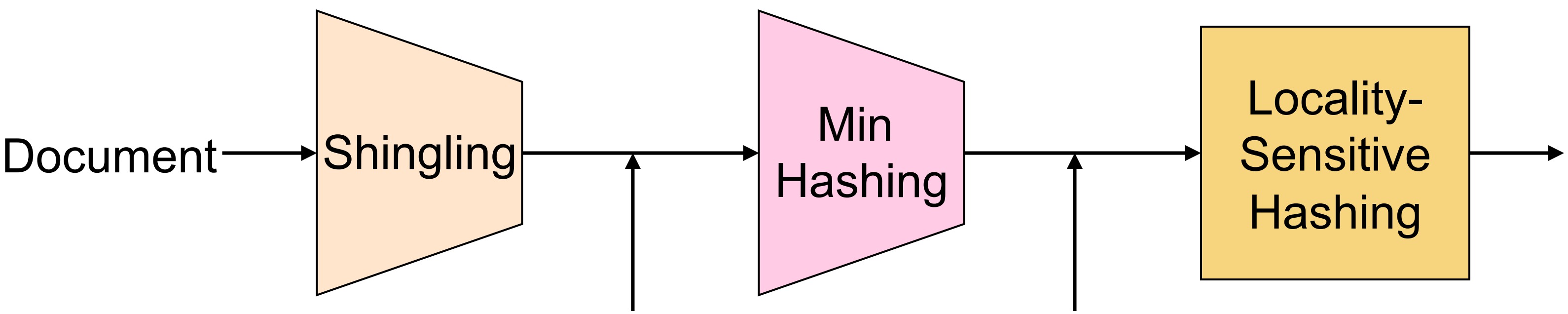
Task: Finding Similar Documents

- **Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs**
- **Applications:**
  - Similar news articles at many news sites
    - Cluster articles by "same story"
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
- **Problems:**
  - Too many documents to compare all pairs
  - Documents are so large or so many that they cannot fit in main memory
  - Many small pieces of one document can appear out of order in another

3 Essential Steps for Similar Docs

1. ***Shingling:*** Convert documents to sets
2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity
3. ***Locality-Sensitive Hashing:*** Focus on pairs of signatures likely to be from similar documents
   - **Candidate pairs!**

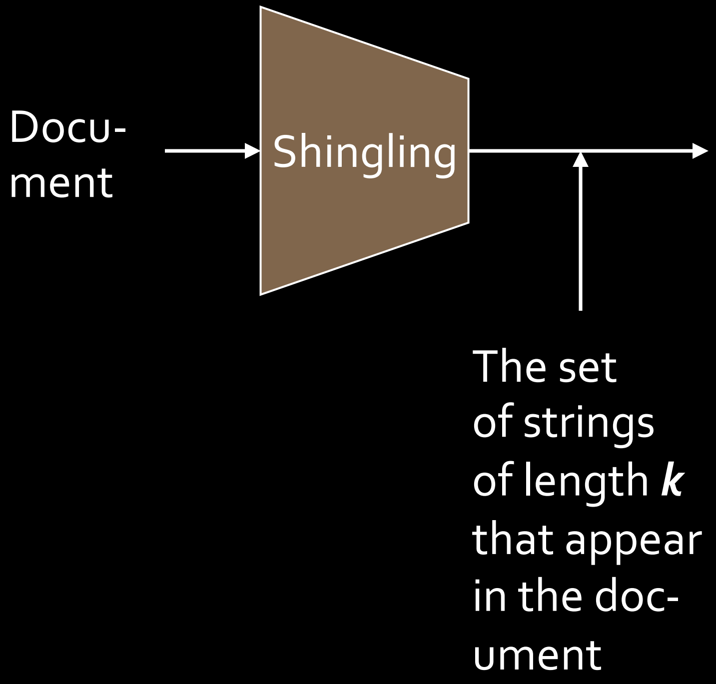
The Big Picture



The set of strings of length ***k*** that appear in the doc-ument

Signatures: short integer vectors that represent the sets, and reflect their similarity

Candidate pairs: those pairs of signatures that we need to test for similarity



Shingling

Step 1: ***Shingling:*** Convert documents to sets

Documents as High-Dim Data

- **Step 1: Convert documents to sets**
- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words (eliminate stop words: "and", "a", "the", "to", "you" and so on)
  - Don't work well for this application. **Why?**

Documents as High-Dim Data

- **Step 1: Convert documents to sets**
- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words (eliminate stop words: "and", "a", "the", "to", "you" and so on)
  - Don't work well for this application. **Why?**
- **Additiotinally need to account for ordering of words!**
  - **Solution**: **Shingles!**

Define: Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

- **Example:** **k=2**; document $\mathbf{D_1}$ = abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  - **Option:** Shingles as a bag (multiset), count ab twice: $\mathbf{S'(D_1)}$ = {ab, bc, ca, ab}

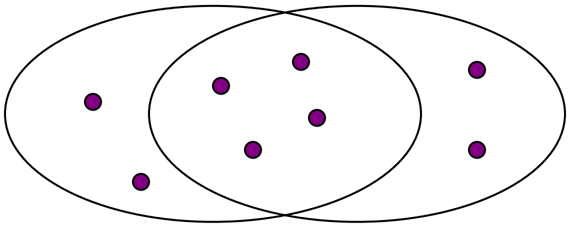
Compressing Shingles

- To **compress long shingles**, we can **hash (or simply map)** them to (say) 4 bytes
- **Represent a document by the set of hash values of its *k*-shingles**
  - **Idea:** Rare (or none) collisions of shingles
- **Example:** **k=2**; document $\mathbf{D_1}$= abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  Hash the singles: $\mathbf{h(D_1)}$ = {1, 5, 7}

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**
- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse
- **A natural similarity measure is the Jaccard similarity:**

$$sim(D_1, D_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$



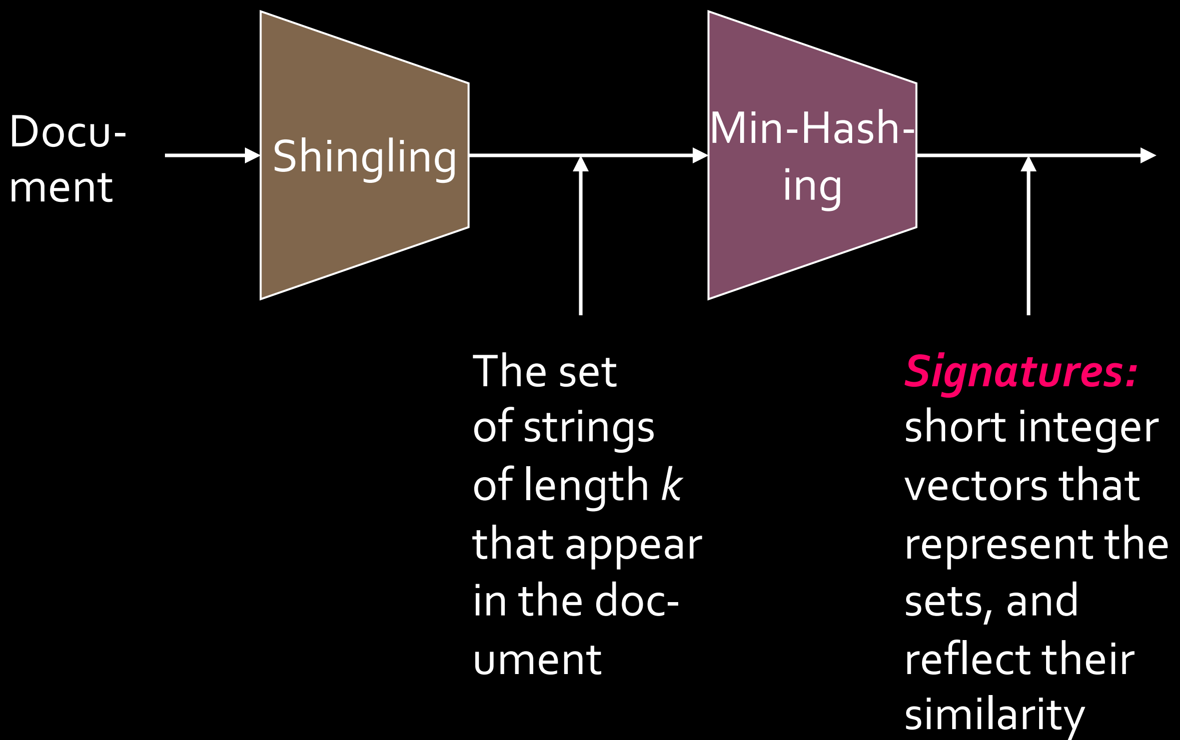
Working Assumption

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- **Caveat:** You must pick ***k*** large enough, or most documents will have most shingles
  - ***k*** = 5 is OK for short documents
  - ***k*** = 10 is better for long documents

Motivation for Minhash/LSH

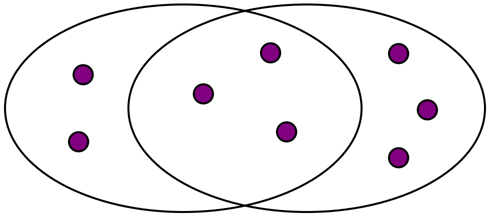
- **Suppose we need to find near-duplicate documents among $N = 1$ million documents**
- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of docs**
  - $\boldsymbol{N(N-1)/2}$ ≈ **5*10^{11}** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **5 days**
- For $N = \mathbf{10}$ million, it takes more than a year…



MinHashing

Step 2: ***Minhashing:*** Convert **large sets** to **short signatures**, while **preserving similarity**

Encoding Sets as Bit Vectors



- Many similarity problems can be formalized as **finding subsets that have significant intersection**
- **Encode sets using 0/1 (bit, boolean) vectors**
  - One dimension per element in the universal set
- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**

- **Example:** $\mathbf{C_1}$ = 10111; $\mathbf{C_2}$ = 10011
  - Size of intersection **= 3**; size of union **= 4,**
  - **Jaccard similarity** (not distance) **= 3/4**
  - **Distance: d(C_1,C_2) = 1 – (Jaccard similarity) = 1/4**

From Sets to Boolean Matrices

- **Rows** = elements (shingles)
- **Columns** = sets (documents)
  - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - **Typical matrix is sparse!**
- **Each document is a column:**
  - **Example: sim(C_1 ,C_2) = ?**
    - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
    - **d(C_1,C_2) = 1 – (Jaccard similarity) = 3/6**

Documents

Shingles

1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0

Outline: Finding Similar Columns

- **So far:**
  - Documents $\rightarrow$ Sets of shingles
  - Represent sets as boolean vectors in a matrix
- **Next goal: Find similar columns while computing small signatures**
  - **Similarity of columns == similarity of signatures**

Outline: Finding Similar Columns

- **Next Goal: Find similar columns, Small signatures**
- **Approach:**
  - **1) Signatures of columns:** small summaries of columns
  - **2) Examine pairs of signatures** to find similar columns
    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar
- **Warnings:**
  - Comparing all pairs may take too much time: **Job for LSH**
    - These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures)

- **Key idea:** "hash" each column C to a small ***signature*** $h(C)$, such that:
  - **(1)** $h(C)$ is small enough that the signature fits in RAM
  - **(2)** $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$

- **Goal: Find a hash function $h(\cdot)$ such that:**
  - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - If $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- **Hash docs into buckets. Expect that "most" pairs of near duplicate docs hash into the same bucket!**

Min-Hashing

- **Goal: Find a hash function $h(\cdot)$ such that:**
  - if $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$
- **Clearly, the hash function depends on the similarity metric:**
  - Not all similarity metrics have a suitable hash function
- **There is a suitable hash function for the Jaccard similarity:** It is called **Min-Hashing**

Min-Hashing

- Imagine the rows of the boolean matrix permuted under **random permutation** π
- Define a **"hash" function** $h_\pi(C)$ = the index of the **first** (in the permuted order π) row in which column C has value **1**:
- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Min-Hashing Example

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

The Min-Hash Property

- **Choose a random permutation π**
- **<u>Claim:</u>** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$

  $= |C1 \cap C2| / |C1 \cup C2|$

0	0
0	0
1	1
0	0
0	1
1	0

Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- **The *similarity of two signatures* is the fraction of the hash functions in which they agree**
- **Note:** Because of the Min-Hash property, the similarity of columns is “almost the same” as the expected similarity of their signatures

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

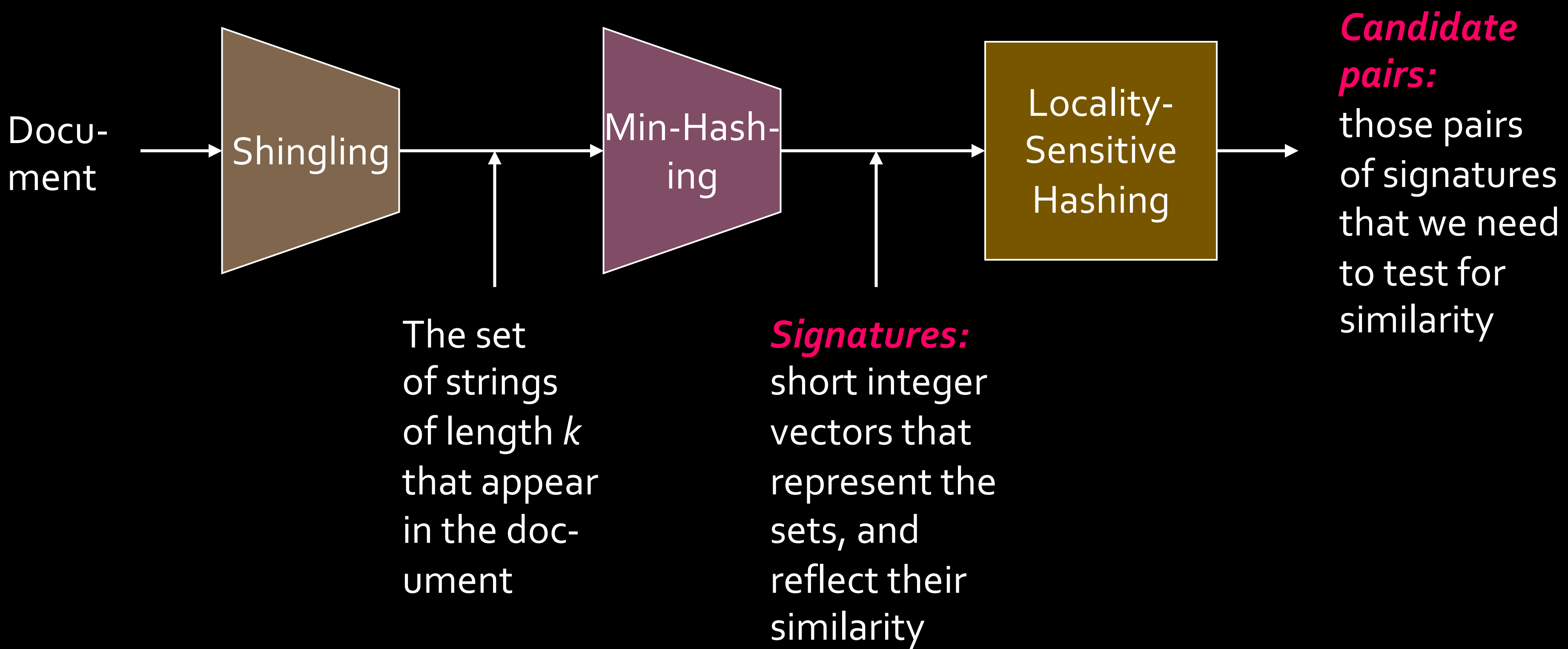
	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
- ***sig*(C)[i] =** according to the *i*-th permutation, the index of the first row that has a 1 in column *C*

Note: The sketch (signature) of document *C* is small $\sim$**100 bytes!**

- **We achieved our goal! We "compressed" long bit vectors into short signatures**



Locality Sensitive Hashing

Step 3: ***Locality-Sensitive Hashing:***
Focus on pairs of signatures likely to be from similar documents

LSH: First Cut

2	1	4	1
1	2	1	2
2	1	2	1

- **Goal:** Find documents with Jaccard similarity at least ***s*** (for some similarity threshold, e.g., ***s***=0.8)
- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***x*** and ***y*** is a ***candidate pair:*** a pair of elements whose similarity must be evaluated
- **For Min-Hash matrices:**
  - Hash columns of signature matrix ***M*** to many buckets
  - Each pair of documents that hashes into the same bucket is a **candidate pair**

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Pick a similarity threshold *s* (0 < s < 1)**
- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree on at least fraction ***s*** of their rows:
  $\boldsymbol{M(i, x) = M(i, y)}$ for at least frac. ***s*** values of ***i***
  - We expect documents ***x*** and ***y*** to have the same (Jaccard) similarity as their signatures

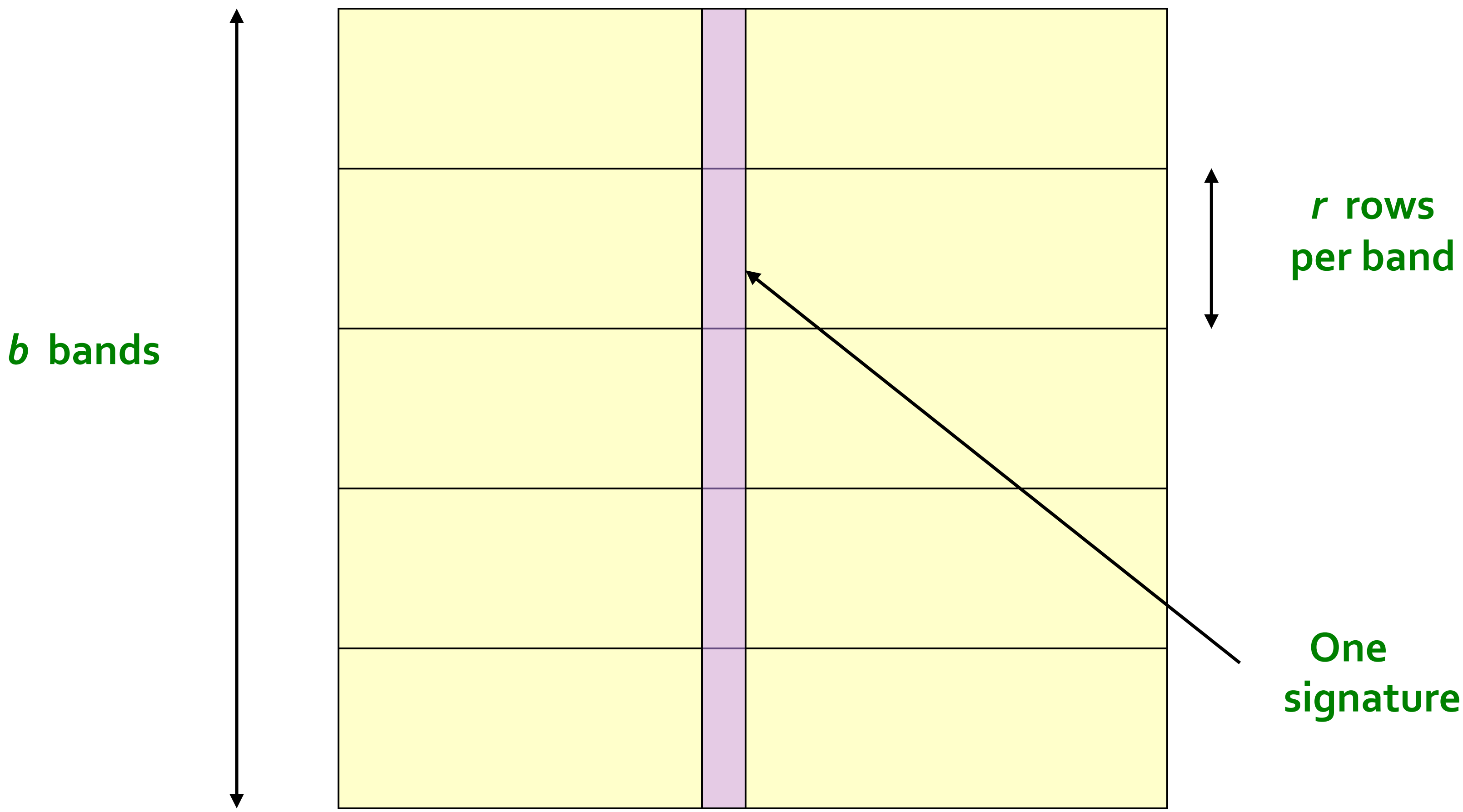
LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- **Big idea: Hash columns of signature matrix *M* several times**
- Arrange that (only) **similar columns** are likely to **hash to the same bucket**, with high probability
- **Candidate pairs are those that hash to the same bucket**

Partition M into b Bands

2	1	4	1
1	2	1	2
2	1	2	1

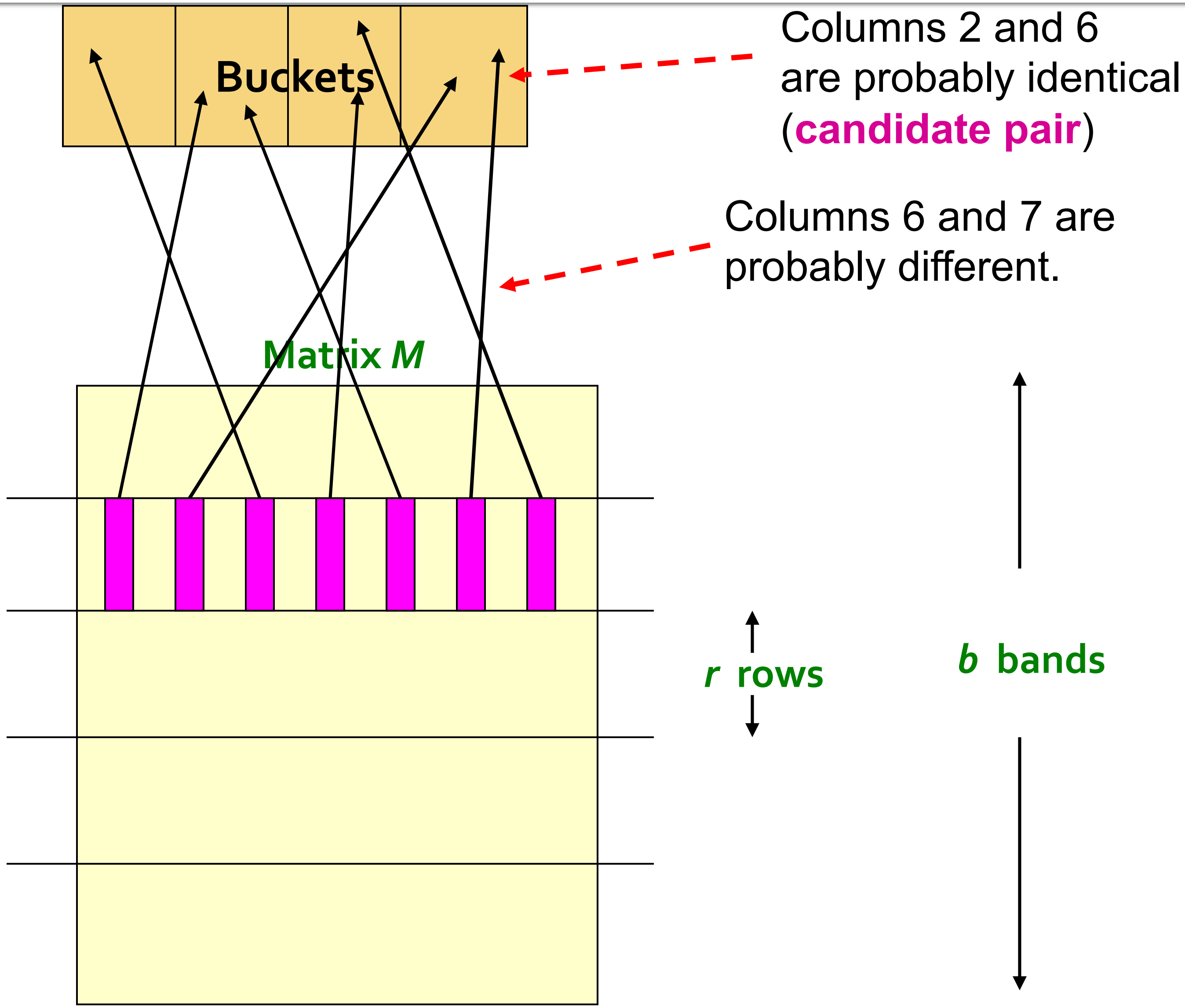


Signature matrix *M*

Partition M into Bands

- Divide matrix ***M*** into ***b*** bands of ***r*** rows
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible
- ***Candidate*** column pairs are those that hash to the same bucket for ≥ **1** band
- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs

Hashing Bands



Simplifying Assumption

- There are **enough buckets** that columns are unlikely to hash to the same bucket unless they are **identical** in a particular band
- Hereafter, we assume that "**same bucket**" means "**identical in that band**"
- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

2	1	4	1
1	2	1	2
2	1	2	1

Assume the following case:

- Suppose 100,000 columns of ***M*** (100k docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take 40Mb
- Choose ***b*** = 20 bands of ***r*** = 5 integers/band

- **Goal:** Find pairs of documents that are at least ***s*** *= 0.8* similar

C_1, C_2 are 80% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- **Find pairs of ≥ *s*=**0.8 similarity, set **b=**20, **r=**5
- **Assume:** sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) ≥ **s**, we want C_1, C_2 to be a **candidate pair**: We want them to hash to at **least 1 common bucket** (at least one band is identical)
- **Probability C_1, C_2 identical in one particular band:** $(0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands: $(1-0.328)^{20} = 0.00035$
  - i.e., about 1/3000th of the 80%-similar column pairs are **false negatives** (we miss them)
  - **We would find 99.965% pairs of truly similar documents**

C_1, C_2 are 30% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- **Find pairs of ≥ *s*=0.8 similarity, set b=20, r=5**
- **Assume:** $sim(C_1, C_2) = 0.3$
  - Since $sim(C_1, C_2) <$ **s** we want C_1, C_2 to hash to **NO common buckets** (all bands should be different)
- **Probability C_1, C_2 identical in one particular band:** $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
    - They are **false positives** since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold **s**

LSH Involves a Tradeoff

2	1	4	1
1	2	1	2
2	1	2	1

- **Pick:**
  - The number of Min-Hashes (rows of ***M***)
  - The number of bands ***b***, and
  - The number of rows ***r*** per band

  to balance false positives/negatives

- **Example:** If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

b bands, *r* rows/band

- Columns C_1 and C_2 have similarity t
- Pick any band (r rows)
  - Prob. that all rows in band equal = t^r
  - Prob. that some row in band unequal = $1 - t^r$
- Prob. that no band identical = $(1 - t^r)^b$
- Prob. that at least 1 band identical =

$$1 - (1 - t^r)^b$$

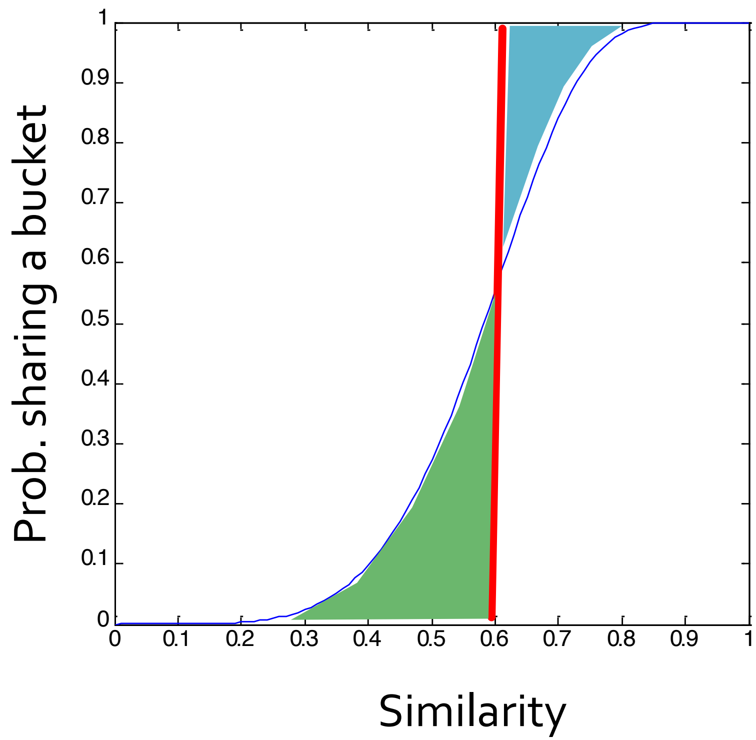
Example: $b = 20$; $r = 5$

- Similarity threshold s
- Prob. that at least 1 band is identical:

s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

Picking *r* and *b*: The S-curve

- **Picking *r* and *b* to get the best S-curve**
  - 50 hash-functions (r=5, b=10)



Blue area: False Negative rate
Green area: False Positive rate

LSH Summary

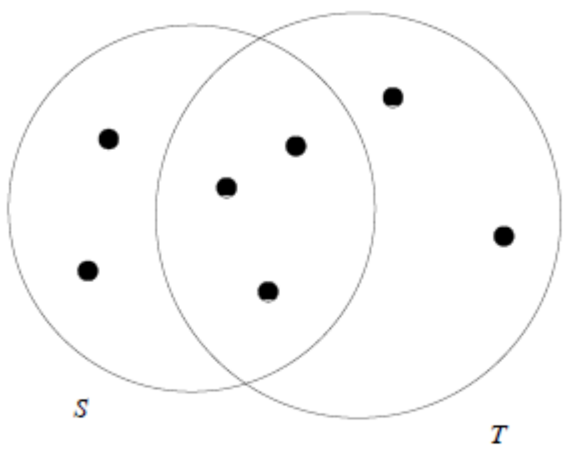
- Tune ***M, b, r*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures
- Check in main memory that **candidate pairs** really do have **similar signatures**
- **Optional:** In another pass through data, check that the remaining candidate pairs really represent similar documents

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Quiz: Jaccard Similarity

- There are two sets S and T in the figure below. What is their Jaccard similarity?



Quiz: Shingles

- Assume we use k = 9 shingles. Is there some lexicographical similarity in the sentences:
  - “The plane was ready for touch down”
  - “The quarterback scored a touchdown”
- How about if we eliminate white spaces?

Quiz: Shingles $k = 2$

- Our corpus of documents is emails. Assume we choose $k = 2$ shingles. How is it going to affect emails similarity?
- What would be your recommendation for k if corpus of documents is emails?

Quiz: Minhashing

- The matrix representing four sets $S_1, \ldots, S_4$ is presented below. Suppose we pick the permutation of rows *beadc*. What is the value of minhash function h(S_i)?

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

Quiz: Minhashing

- The matrix representing four sets $S_1, \ldots, S_4$ is presented below. Suppose we pick the permutation of rows *beadc*. What is the value of minhash function h(S_i)?

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

Element	S_1	S_2	S_3	S_4
b	0	0	1	0
e	0	0	1	0
a	1	0	0	1
d	1	0	1	1
c	0	1	0	1

Quiz: Permutations

- The matrix representing four sets $S_1, \ldots, S_4$ is presented below. Suppose, we pick the permutation h_1 = x + 1 mod 5 and 3x + 1 mod 5.
  - Compute permutation hash functions for the matrix based on the row.

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1		
1	0	0	1	0		
2	0	1	0	1		
3	1	0	1	1		
4	0	0	1	0		

Quiz: Computing the signature Matrix

- Compute the signature matrix with **single pass** over two permutations established by the hash functions in the previous task.

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

Quiz: Jaccard over the Signature matrix

- Estimate the Jaccard similarities of the underlying sets S_1 and S_4 from the signature matrix.

	S_1	S_2	S_3	S_4
h_1	1	3	0	1
h_2	0	2	0	0

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x+1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

- What is the actual Jaccard similarity between S_1 and S_4?

Quiz: LSH

- Evaluate the S-curve $1-(1-s^r)^b$ for s = 0.5 and 0.8, for the following values of r and b:
  - r = 3 and b = 10.

Summary: Jaccard Similarity

- The Jaccard similarity of sets is the ratio of the size of the intersection of the sets to the size of the union.
- This measure of similarity is suitable for many applications, including textual similarity of documents and similarity of buying habits of customers.

Summary: Shingling

- A k-shingle is any k characters that appear consecutively in a document.
- If we represent a document by its set of k-shingles, then the Jaccard similarity of the shingle sets measures the textual similarity of documents.
- Sometimes, it is useful to hash shingles to bit strings of shorter length, and use sets of hash values to represent documents.

Summary: Minhashing

- A minhash function on sets is based on a permutation of the universal set.
- Given any such permutation, the minhash value for a set is that element of the set that appears first in the permuted order.

Summary: Efficient Minhashing

It is normal to simulate a permutation by

- picking a random hash function and
- taking the minhash value for a set to be the least hash value of any of the set's members..

Summary: LSH

- Locality sensitive hashing technique allows us to avoid computing the similarity of every pair of sets or their minhash signatures.
- If we are given signatures for the sets, we may divide them into bands, and only measure the similarity of a pair of sets if they are identical in at least one band.
- By choosing the size of bands appropriately, we can eliminate from consideration most of the pairs that do not meet our threshold of similarity.

Actions

- Finish Quiz unless you did over the lecture.
- Review slides!
- Read Chapter 3 from course book.
  - You can find electronic version of the book on Blackboard.