Frequent Itemset Mining & Association Rules

Big Data Analytics CSCI 4030

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If someone buys diaper and milk, then he/she is likely to buy beer
  - Don't be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A large set of **items**
  - e.g., things sold in a supermarket
- A **large set** of **baskets**
- Each basket is a **small subset of items**
  - e.g., the things one customer buys on one day
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!

Input:

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Chain stores keep data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer
  - Need the rule to occur frequently, or no $$'s
- ***Amazon's people who bought X also bought Y***

Applications – (2)

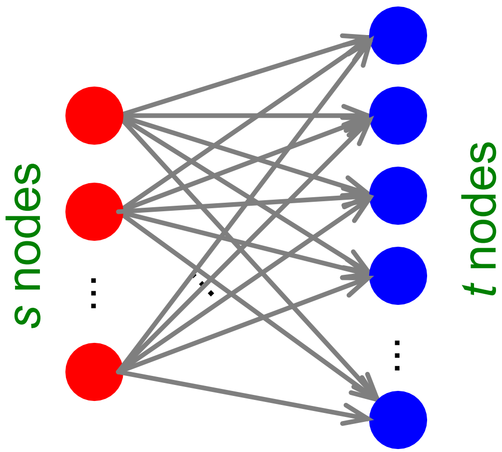
- **Baskets** = sentences; **Items** = documents containing those sentences
  - Items that appear together too often could represent plagiarism
- **Baskets** = patients; **Items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects

More generally

- **A general many-to-many mapping (association) between two kinds of things**
  - But we ask about connections among “items”, not “baskets”
- **For example:**
  - Finding communities in graphs (e.g., Twitter)

Example:

- **Finding communities in graphs (e.g., Twitter)**
- **Baskets** = nodes; **Items** = outgoing neighbors
  - Searching for complete bipartite subgraphs of a big graph



A dense 2-layer graph

Outline

First: Define

- **Frequent itemsets**
- **Association rules:**
  - Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- **Finding frequent pairs**
- **A-Priori algorithm**
- **PCY algorithm + 2 refinements**

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- ***Support*** for itemset I: Number of baskets containing all items in I
  - (Often expressed as a fraction of the total number of baskets)
- Given a ***support threshold*** s, then sets of items that appear in at least s baskets are called ***frequent itemsets***

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Support threshold** = 3 baskets

  B_1 = {m, c, b} **B_2** = {m, p, j}
  B_3 = {m, b} **B_4** = {c, j}
  B_5 = {m, p, b} **B_6** = {m, c, b, j}
  B_7 = {c, b, j} **B_8** = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j}, {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- **In practice there are many rules, want to find significant/interesting ones!**
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \rightarrow \boldsymbol{milk}$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence will be high
- **Interest** of an association rule $I \rightarrow j$: difference between its confidence and the fraction of baskets that contain j

$$\text{Interest}(I \rightarrow j) = \text{conf}(I \rightarrow j) - \Pr[j]$$

  - Interesting rules are those with high interest values (usually above 0.5)

Example: Confidence and Interest

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, b\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Association rule: {m, b} → c**
  - **Confidence** = 2/4 = 0.5
  - **Interest** = |0.5 – 5/8| = 1/8
    - Item ***c*** appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Association Rules

- **Problem: Find all association rules with support $\geq s$ and confidence $\geq c$**
- **Hard part: Finding the frequent itemsets!**

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2: Rule generation**
  - For every subset A of I, generate a rule $A \to I \setminus A$
    - Since I is frequent, A is also frequent
    - Single pass to compute the rule confidence
      - confidence(***A,B***→***C,D***) = support(**A,B,C,D**) / support(**A,B**)
  - **Output the rules above the confidence threshold**

Example

B_1 = {m, c, b}
B_2 = {m, p, j}
B_3 = {m, c, b, n}
B_4= {c, j}
B_5 = {m, p, b}
B_6 = {m, c, b, j}
B_7 = {c, b, j}
B_8 = {b, c}

- **Support threshold $s = 3$, confidence $c = 0.75$**
- **1) Frequent itemsets:**
  - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- **2) Generate rules:**
  - ~~b→m: c=4/6~~ b→c: c=5/6 ~~b,c→m: c=3/5~~
  - m→b: c=4/5 … b,m→c: c=3/4
  - ~~b→c,m: c=3/6~~

Compacting the Output

- **To reduce the number of rules we can post-process them and only output:**
  - **Maximal frequent itemsets:**
    No immediate superset is frequent
    - Gives more pruning

Example: Maximal/Closed

	Support	Maximal(s=3)
A	4	No
B	5	No
C	3	No
AB	4	Yes
AC	2	No
BC	3	Yes
ABC	2	No

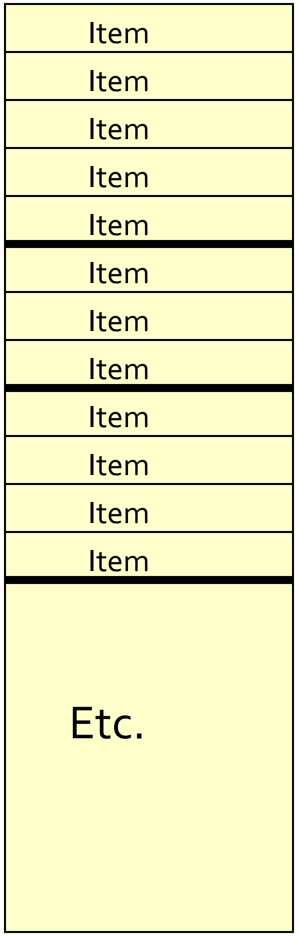
Frequent, but superset BC also frequent. (→ B: No)

Frequent, and its only superset, ABC, not freq. (→ AB: Yes)

Finding Frequent Itemsets

Itemsets: Computation Model

- **Back to finding frequent itemsets**
- Data is often kept in flat files rather than in a database system:
  - Stored on disk
  - Stored basket-by-basket
  - Baskets are **small** but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use ***k*** nested loops to generate all sets of size ***k***



Items are positive integers, and boundaries between baskets are –1.

Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Computation Model

- The true cost of mining disk-resident data is usually the **number of disk I/Os**
- In practice, association-rule algorithms read the data in ***passes*** – all baskets read in turn
- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs of items
  - The number of different things we can count is limited by main memory
  - Swapping in/out is a disaster (why?)

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs of items $\{i_1, i_2\}$**
  - **Why?** Freq. pairs are common, freq. triples are rare
    - **Why?** Probability of being frequent drops exponentially with size;
- **Let's first concentrate on pairs, then extend to larger sets**
- **The approach:**
  - We always need to generate all the itemsets
  - But we would only like to count (keep track) of those itemsets that in the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
  - From each basket of ***n*** items, generate its ***n(n-1)/2*** pairs by two nested loops
- **Fails if $(\#items)^2$ exceeds main memory**
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)
    - Suppose 10^5 items, counts are 4-byte integers
    - Number of pairs of items: $10^5(10^5\text{-}1)/2 = 5*10^9$
    - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

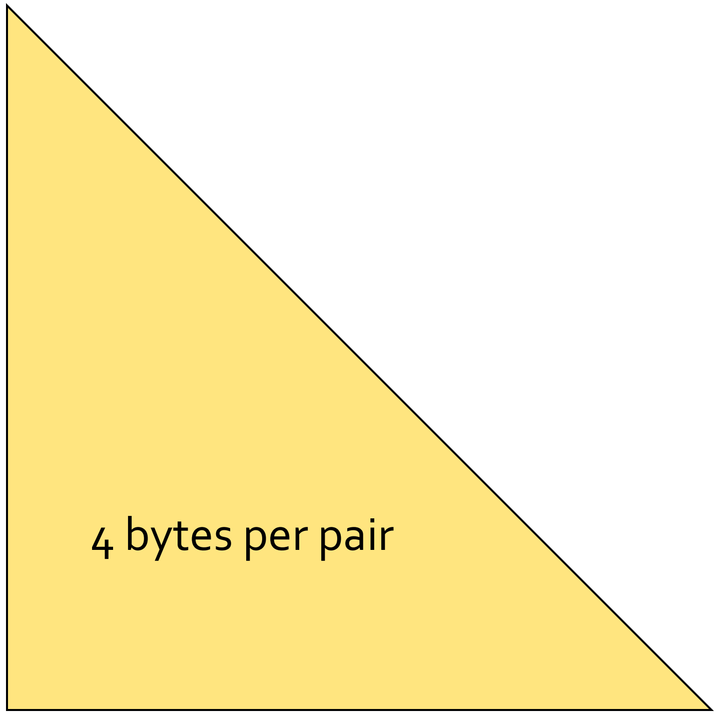
Two approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep triples [*i*, *j*, *c*] = "the count of the pair of items {*i*, *j*} is *c*."
  - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with **count > 0**

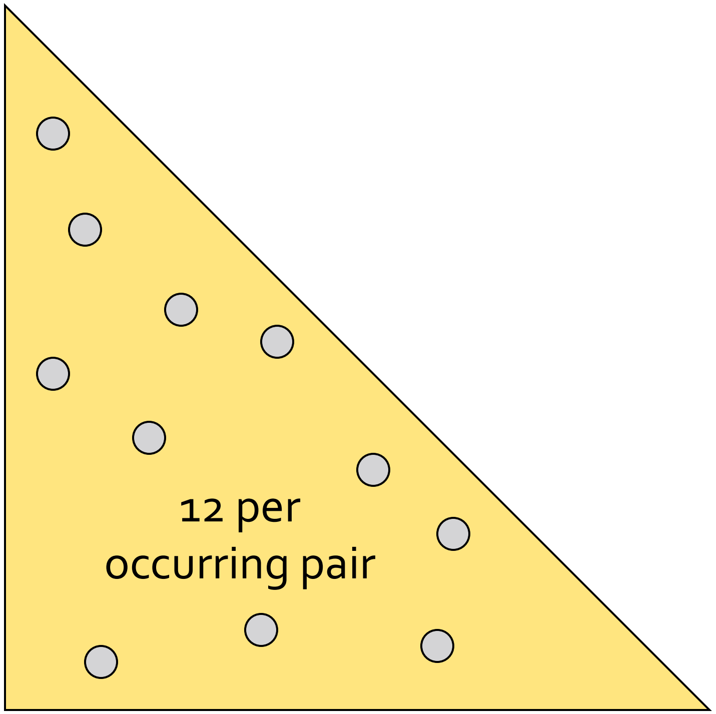
Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches



Triangular Matrix



Triples

Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Count pair of items $\{i, j\}$ only if $i<j$
  - Keep pair counts in lexicographic order:
    - $\{1,2\}, \{1,3\},\ldots, \{1,n\}, \{2,3\}, \{2,4\},\ldots,\{2,n\}, \{3,4\},\ldots$
  - Pair $\{i, j\}$ is at position $(i-1)(n-i/2) + j-1$
  - Total number of pairs $\boldsymbol{n(n-1)/2}$; total bytes= $\boldsymbol{2n^2}$
  - **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses **12 *bytes*** per occurring pair *(but only for pairs with count > 0)*
  - Beats Approach 1 if less than **1/3** of possible pairs actually occur

Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Co
  - K
  - F
  - T ... $2n^2$
  - **T**
- **Ap**
  (bu
  - Beats Approach 1 if less than **1/3** of possible pairs actually occur

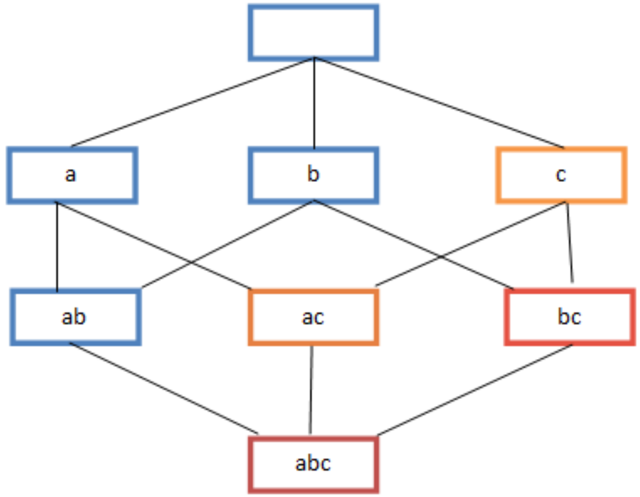
Problem is if we have too many items so the pairs do not fit into memory.

Can we do better?

A-Priori Algorithm

A-Priori Algorithm – (1)

- A **two-pass** approach called ***A-Priori*** limits the need for main memory



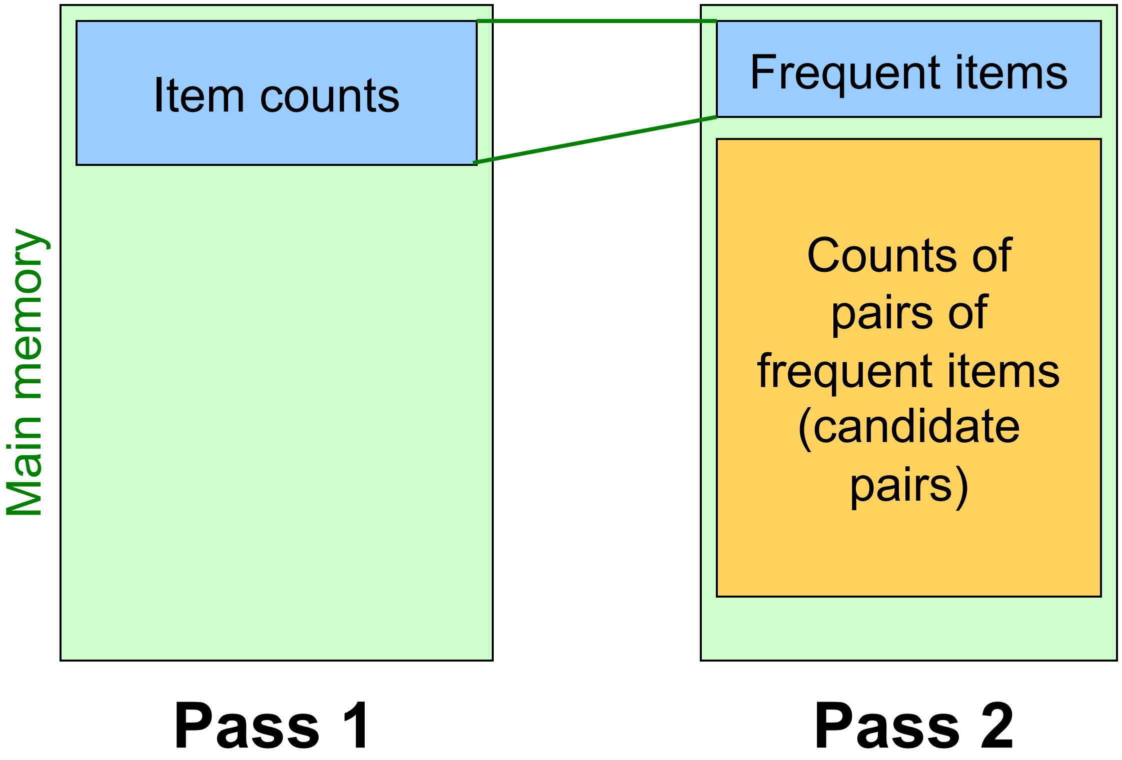
- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every **subset** J of I
- **Contraprositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets

- **So, how does A-Priori find freq. pairs?**

A-Priori Algorithm – (2)

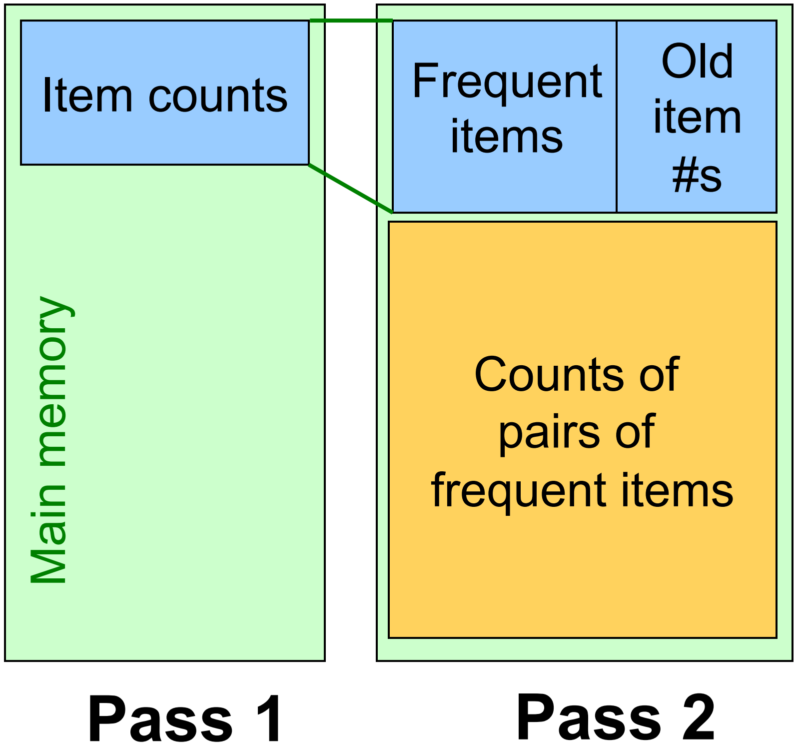
- **Pass 1:** Read baskets and count in main memory the occurrences of each **individual item**
  - Requires only memory proportional to #items
- **Items that appear $\geq s$ times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



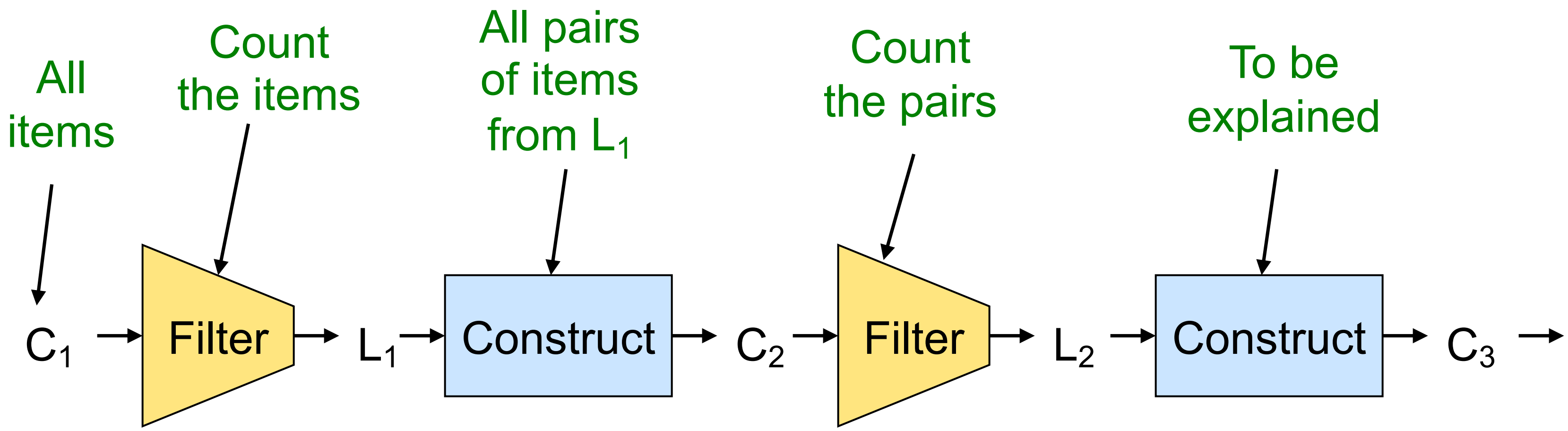
Detail for A-Priori

- You can use the triangular matrix method with ***n*** = number of frequent items
  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct**
  - ***C_k*** = ***candidate*** = those that might be frequent sets (support ≥ **s**) based on information from the pass for ***k*–1**
  - ***L_k*** = the set of truly frequent ***k***-tuples



Example

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1.

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} {b,c,j} {b,m,j} {c,m,j} } **
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }

A-Priori for All Frequent Itemsets

- One pass for each ***k*** (itemset size)
- Needs room in main memory to count each candidate
- For typical market-basket data and reasonable support (e.g., 1%), ***k* = 2** requires the most memory

- **Many possible extensions:**
  - Association rules with intervals:
    - For example: Men over 50 have 2 cars
  - Association rules when items are in a taxonomy
    - Bread, Butter $\rightarrow$ FruitJam
  - Lower the support ***s*** as itemset gets bigger

PCY (Park-Chen-Yu) Algorithm

Park-Chen-Yu

- **Observation:**
  In pass 1 of A-Priori, most memory is idle
  - We store only individual item counts
  - **Can we use the idle memory to reduce memory required in pass 2?**
- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory
  - Keep a **count** for each bucket into which **pairs** of items are hashed
    - **For each bucket just keep the count, not the actual pairs that hash to the bucket!**

PCY Algorithm – First Pass

New in PCY (applies to the "FOR (each pair of items)" block)

```
FOR (each basket) :
    FOR (each item in the basket) :
        add 1 to item's count;
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

- **Few things to note:**
  - Pairs of items need to be generated from the input file; they are not present in the file
  - We are not just interested in the presence of a pair, but we need to see whether it is present at least ***s*** (support) times

Observations about Buckets

- **Observation: If a bucket contains a frequent pair, then the bucket is surely frequent**
- However, even without any frequent pair, a bucket can still be frequent ☹
- **But, for a bucket with total count less than s, none of its pairs can be frequent ☺**
  - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- **Pass 2:**
  Only count pairs that hash to frequent buckets

PCY Algorithm – Between Passes

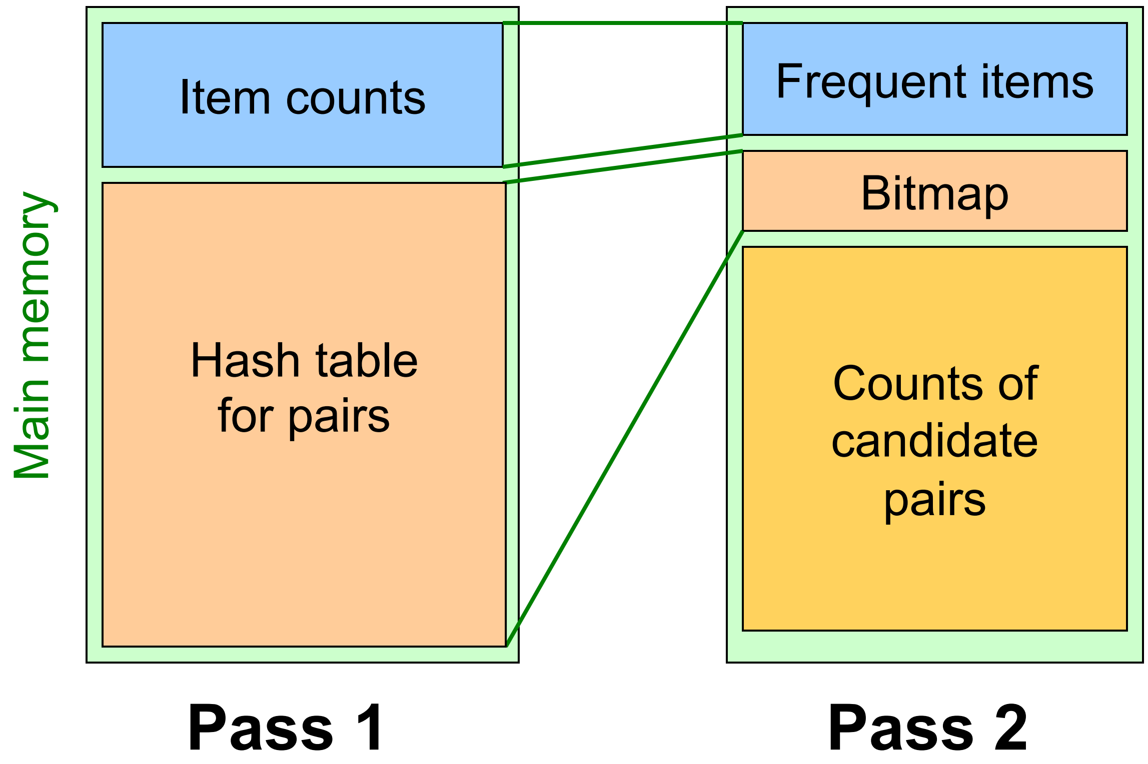
- **Replace the buckets by a bit-vector:**
  - **1** means the bucket count exceeded the support s (call it a **frequent bucket**); **0** means it did not
- **4-byte integer counts are replaced by bits, so the bit-vector requires 1/32 of memory**
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs *{i, j}* that meet the conditions for being a **candidate pair**:
  1. Both *i* and *j* are frequent items
  2. The pair *{i, j}* hashes to a bucket whose bit in the bit vector is **1** (i.e., a **frequent bucket**)

- **Both conditions are necessary for the pair to have a chance of being frequent!**

Main-Memory: Picture of PCY



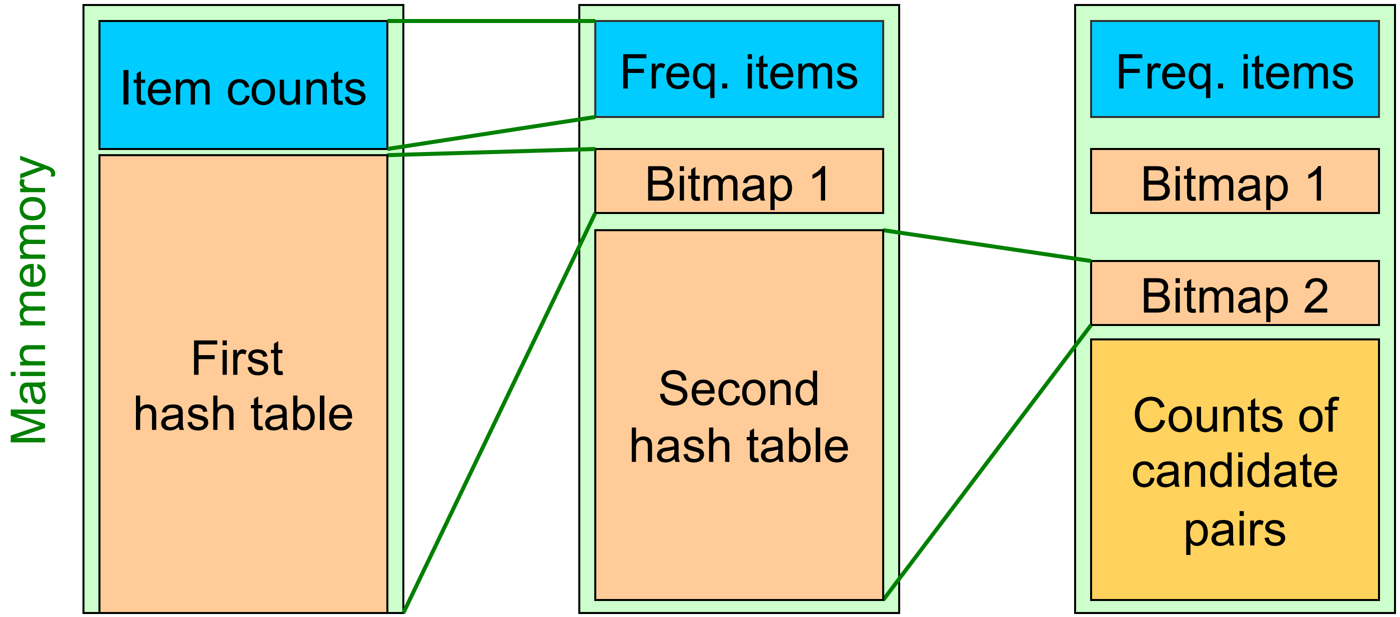
Main-Memory Details

- **Buckets require a few bytes each:**
  - #buckets is *O(main-memory size)*
- On second pass, a table of (item, item, count) triples is essential
  - Hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat A-Priori

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
  - **Remember:** Memory is the bottleneck
  - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
  - ***i*** and ***j*** are frequent, and
  - ***{i, j}*** hashes to a frequent bucket from **Pass 1**
- On middle pass, fewer pairs contribute to buckets, so fewer ***false positives***
- **Requires 3 passes over the data**

Main-Memory: Multistage



Pass 1

Count items
Hash pairs {i,j}

Pass 2

Hash pairs {i,j}
into Hash2 iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1

Pass 3

Count pairs {i,j} iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1
{i,j} hashes to
freq. bucket in B2

Multistage – Pass 3

- **Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:**
  1. Both i and j are frequent items
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is **1**
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is **1**

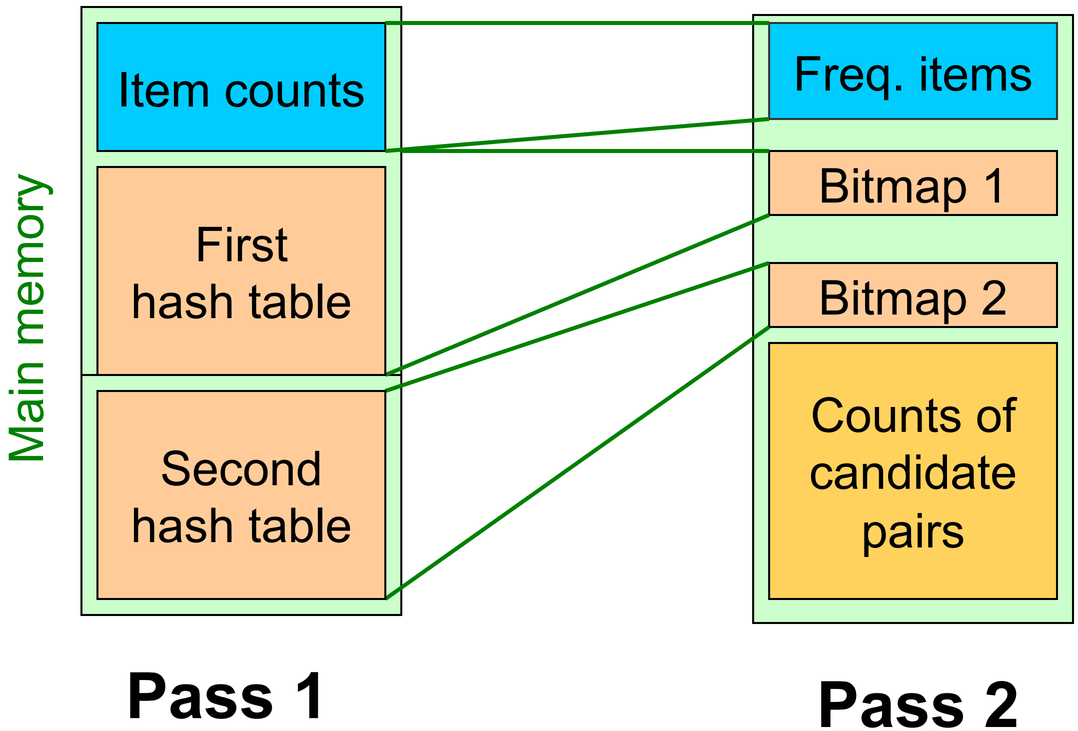
Important Points

1. **The two hash functions have to be independent**
2. **We need to check both hashes on the third pass**
    - If not, we would end up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes

Main-Memory: Multihash



PCY: Extensions

- Either **multistage** or **multihash** can use more than two hash functions
- In **multistage**, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
- For **multihash**, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$

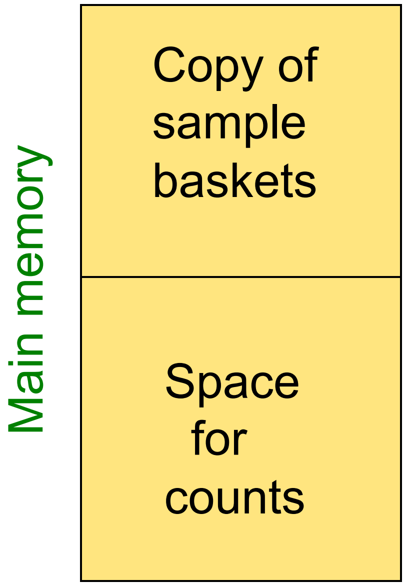
Frequent Itemsets in $\leq$ 2 Passes

Frequent Itemsets in ≤ 2 Passes

- A-Priori, PCY, etc., take k passes to find frequent itemsets of size k
- **Can we use fewer passes?**
- Use 2 or fewer passes for all sizes, but may miss some frequent itemsets
  - Random sampling
  - SON (Savasere, Omiecinski, and Navathe)
  - Toivonen (see textbook)

Random Sampling (1)

- Take a random sample of the market baskets
- Run a-priori or one of its improvements in main memory
  - So we don't pay for disk I/O each time we increase the size of itemsets
  - Reduce support threshold proportionally to match the sample size



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)

- But you don’t catch sets frequent in the whole but not in the sample
  - Smaller threshold, e.g., $s/125$, helps catch more truly frequent itemsets
    - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
  - Note: we are not sampling, but processing the entire file in memory-sized chunks
- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set

- **Key "monotonicity" idea:** an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON – Distributed Version

- SON lends itself to distributed data mining
- Baskets distributed among many nodes
  - Compute frequent itemsets at each node
  - Distribute candidates to all nodes
  - Accumulate the counts of all candidates

Summary

- Market-Basket model of data assumes there are two kinds of entities: items and baskets. There is a many–many relationship between items and baskets.
- Typically, baskets are related to small sets of items, while items may be related to many baskets.

Summary

- The support for a set of items is the number of baskets containing all those items.
  - Itemsets with support that is at least some threshold are called frequent itemsets.
- Association Rules: These are implications that if a basket contains certain set of items I, then it is likely to contain another particular item j as well.
  - The probability that j is also in a basket containing I is called the confidence of the rule.
  - The interest of the rule is the amount by which the confidence deviates from the fraction of all baskets that contain j.

Summary

- Monotonicity of Frequent Itemsets: An important property of itemsets is that if a set of items is frequent, then so are all its subsets.
- We exploit this property to eliminate the need to count certain itemsets by using its Contrapositive.
- A-Priori algorithm allows us to find frequent itemsets larger than pairs, if we make one pass over the baskets for each size itemset, up to some limit.
- To find the frequent itemsets of size k, monotonicity lets us restrict our attention to only those itemsets such that all their subsets of size k – 1 have already been found frequent.

Summary

- The Multistage Algorithm: We can insert additional passes between the first and second pass of the PCY Algorithm to hash pairs to other, independent hash tables.
- At each intermediate pass, we only have to hash pairs of frequent items that have hashed to frequent buckets on all previous passes.

Summary

- The Multihash Algorithm: We can modify the first pass of the PCY Algorithm to divide available main memory into several hash tables.
- On the second pass, we only have to count a pair of frequent items if they hashed to frequent buckets in all hash tables.
- Alternatives:
  - Randomized Algorithms (Sampling)
  - The SON Algorithm (Segments)

Quiz: Frequent Itemsets

- Consider following set of baskets. Assume we set our threshold at $s = 3$. Compute frequent pairs of items.
  1. {Cat, and, dog, bites}
  2. {Yahoo, news, claims, a, cat, mated, with, a, dog, and, produced, viable, offspring}
  3. {Cat, killer, likely, is, a, big, dog}
  4. {Professional, free, advice, on, dog, training, puppy, training}
  5. {Cat, and, kitten, training, and, behavior}
  6. {Dog, &, Cat, provides, dog, training, in, Eugene, Oregon}
  7. {Dog, and, cat, is, a, slang, term, used, by, police, officers, for, a, male, female, relationship}
  8. {Shop, for, your, show, dog, grooming, and, pet, supplies}

Quiz: Frequent Itemsets

- Are there any frequent triples and quadruples?

Quiz: Association Rules (Confidence)

- Consider the baskets in Slide 64.
  - What is the confidence of the rule: {*cat*, *dog*} → *and*?
  - What is the confidence of the rule: {*cat*} → *kitten*?

Quiz: Association Rules (Interest)

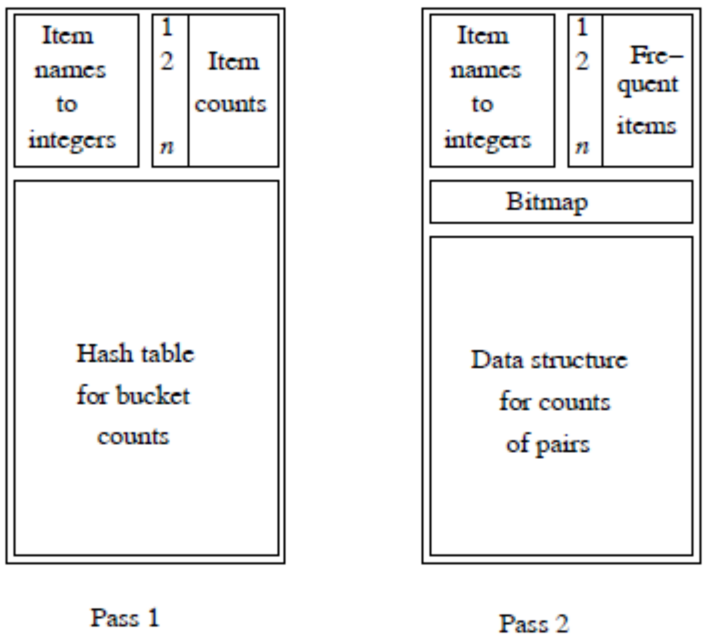
- Consider the baskets in Slide 64.
  - What is the interest of the rule: {*dog*} → *cat*?
  - What is the interest of the rule: {*cat*} → *kitten*?
  - Are these rules “interesting”?

Quiz: Apriori Algorithm

- Compute frequent itemsets for the baskets below with Apriori Algorithm. Assume threshold $s = 3$.
  a) {1, 2, 4, 5, 8, 9}
  b) {1, 4, 7, 8, 9}
  c) {1, 2, 5, 9}
  d) {1, 2, 5, 7, 8}

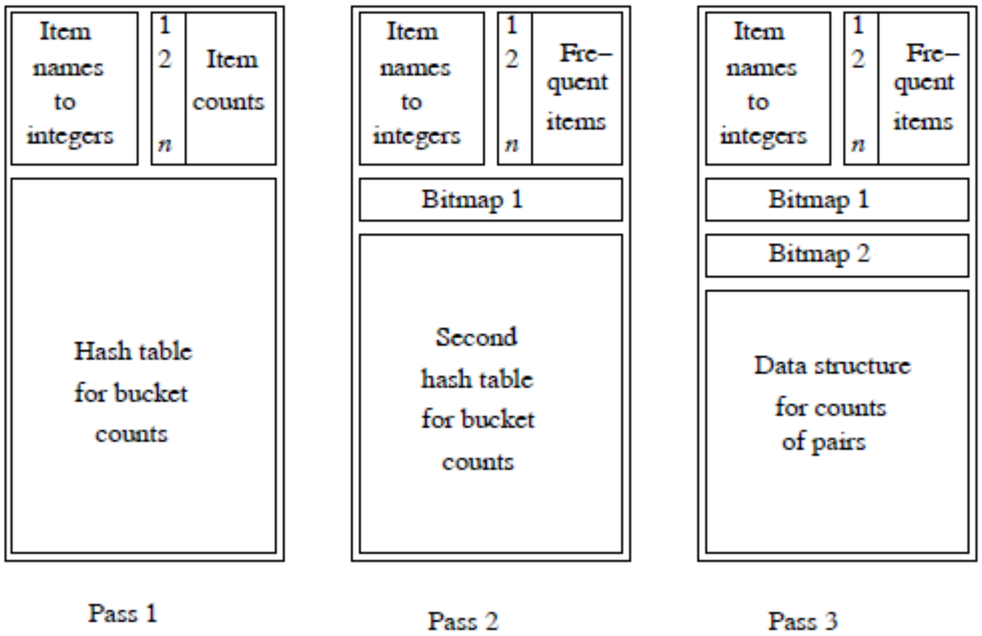
Quiz: PCY algorithm

- Describe how the bitmap is used in PCY algorithm.
- Why is the hash map in main memory from Pass 1 transformed into a bitmap in PCY algorithm?



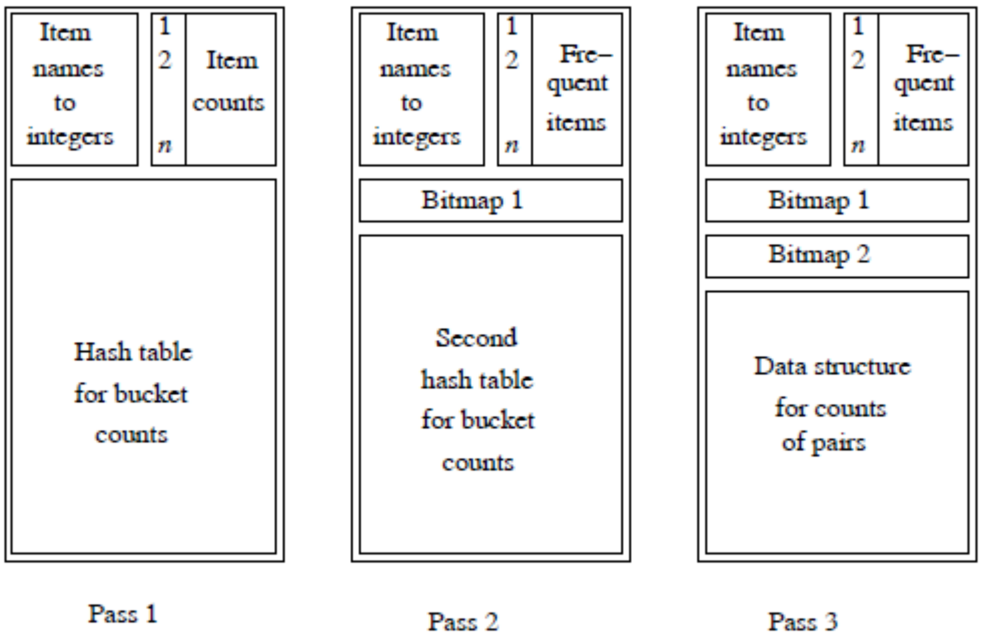
Quiz: Multistage Algorithm

- Describe the key idea behind the multistage algorithm



Quiz: # of Passes

- What wrong can potentially happen if instead of 3 passes one will use 100 passes in multistage algorithm?



Actions

- Review slides!
- Read Chapter 6 from course book.
  - You can find electronic version of the book on Blackboard.